

## Exercises on Symmetric TSP

These problems are taken from various sources. Problems marked \* are more difficult but also more fun :).

Also, please do the exercises in any order.

- 1 Show that any graph has an even number of odd-degree vertices.
- 2 For a 2-edge connected cubic graph, show that  $x_e = 1/3$  for all  $e \in E$  is a feasible solution to Edmond's perfect matching polytope.
- 3 Can you find an  $n$ -vertex instance for which there is a tour of length  $n$  but Christofides' algorithm returns a solution of length close to  $3n/2$ ?  
*This shows that the analysis of Christofides' algorithm is tight.*
- 4 (\*) It is well-known that the vertices of a 2-edge connected graph can be ordered  $v_1, v_2, \dots, v_n$  such that for  $i = 2, \dots, n-1$ ,  $v_i$  has a neighbor  $v_k$  to the left (i.e.,  $k < i$ ) and a neighbor  $v_\ell$  to the right (i.e.,  $\ell > i$ ). This is called an  $st$ -numbering of the graph. Can you use the  $st$ -numbering to give an alternative definition of the set  $R$  of edges we wish to remove to prove that any cubic 2-edge connected graph has a connected Eulerian graph with at most  $4n/3 - 2/3$  edges?
- 5 (\*\*\*) Generalize the proof to show that any 2-edge connected subcubic graph has a connected Eulerian graph with at most  $4n/3 - 2/3$  edges.
- 6 Can you think of a good LP relaxation for the traveling salesman problem?