ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Exercises on Asymmetric TSP

These problems are taken from various sources. Problems marked * are more difficult but also more fun :).

Also, please do the exercises in any order although it might be a good idea to do 3 before 4.

1 We say that an ATSP instance is β-balanced if $c_{ij}/\beta \leq c_{ji} \leq \beta c_{ij}$ for each pair i, j of vertices/cities.

Design an $O(\beta)$-approximation algorithm for β-balanced instances.

2 (*) Consider the following algorithm for ATSP:

- Find a directed cycle C in G minimizing $\frac{c(E(C))}{|C|}$ and add the edges of C, i.e., $E(C)$, to the solution.
- Remove all but one of the vertices of C from G and proceed recursively until G is reduced to a single vertex.

Prove that this algorithm is a $O(\ln n)$-approximation algorithm for ATSP.

3 Give a polynomial time algorithm for finding a min cost cycle cover.

Hint: use a bipartite graph in some way.

4 (*) Show that there exists a cycle cover of no higher cost than the optimal value to the LP-relaxation of ATSP.

Hint: use that $\{x \in \mathbb{R}^E_{\geq 0} : x(\delta(v)) = 1 \text{ for } v \in V\}$ defines the perfect matching polytope for bipartite graphs. Can you prove that?

5 Prove the following:

"Given a solution x to the LP relaxation of ATSP, z defined by

$$z_{\{i,j\}} = \frac{n-1}{n}(x_{ij} + x_{ji})$$

is a feasible solution to the MST polytope."